The block AAA algorithm

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1 Introduction

Since version 2.9 the RKToolbox provides two new utility functions `util_block_aaa` and `util_rmse` for working with rational functions represented in a generalized barycentric form with matrix-valued weights,

$$R(z) = \left(\sum_{i=0}^{d} W_i/(z - z_i) \right)^{-1} \left(\sum_{i=0}^{d} W_i F(z_i)/(z - z_i) \right).$$

In this notation, the W_i are $m \times m$ nonsingular weight matrices, the z_i are pairwise distinct barycentric support points in the complex plane, and the $F(z_i)$ are $m \times n$ matrices. Different from the representation of rational functions computed by the RKFIT method [1], the AAA method [5] and its surrogate [2] and set-valued variants [4], the rational function $R(z)$ has a nonscalar "denominator matrix polynomial." See [3] for more details and a number of numerical experiments with different representations of rational matrix-valued functions.

2 Demonstration

We focus on a simple 2×3 matrix-valued function $F(z)$ to demonstrate the use of `util_block_aaa`. This function is defined as below and we sample it at 20 equidistant points on the interval $[0, 10]$:

```
F = @(z) [ 1/(z+1)       1/(z^2-5)              z ;
           1/(z^2+5+1i)  (2+z^2)/(z^3+3*z^2+1)  7 ];
pts = linspace(0,10,20);
```

Together with an error tolerance `tol` and a number of maximal iterations `maxit`, we have all that is needed to reapproximate F using `util_block_aaa`:

```
opts.tol = 1e-12;
opts.maxit = 10;
[R,rmse,out] = util_block_aaa(F,pts,opts);
```

The output `R` is a `baryfun` object and we can display its info as follows:

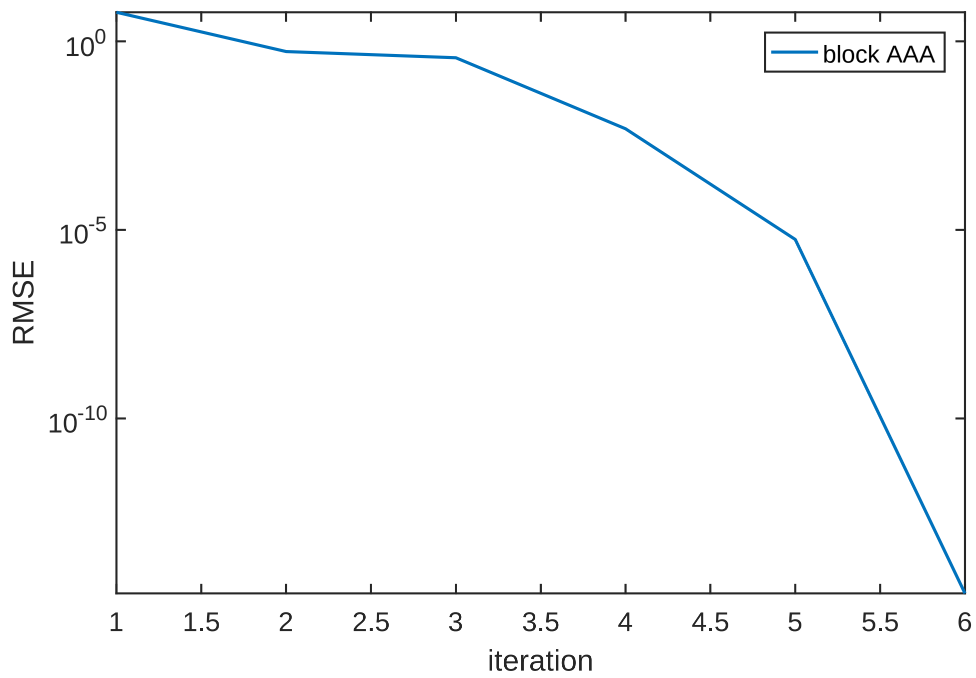
```
disp(R)
```

```
        BARYFUN object of block size 2-by-3 and degree 5.
```

The output `rmse` stores the root mean squared error over all sampling points for each iteration of the block AAA method. In this case, we have resolved F to about machine precision after 6 iterations:

```
semilogy(rmse)
axis tight, hold on
legend('block AAA')
xlabel('iteration'), ylabel('RMSE')
disp(['The final RMSE is ' num2str(util_rmse(pts,F,R)) ])
```

```
The final RMSE is 2.3012e-15
```



3 References

[1] M. Berljafa and S. Güttel. *The RKFIT algorithm for nonlinear rational approximation,* SIAM J. Sci. Comput., 39(5):A2049–A2071, 2017.

[2] S. Elsworth and S. Güttel. *Conversions between barycentric, RKFUN, and Newton representations of rational interpolants,* Linear Algebra Appl., 576:246–257, 2019.

[3] I. V. Gosea and S. Güttel. *Algorithms for the rational approximation of matrix-valued functions,* arXiv preprint 2003.06410v1, 2020. (https://arxiv.org/abs/2003.06410)

[4] P. Lietaert, J. Perez, B. Vandereycken, and K. Meerbergen. *Automatic rational approximation and linearization of nonlinear eigenvalue problems,* arXiv preprint 1801.08622, 2018. (https://arxiv.org/pdf/1801.08622.pdf)

[5] Y. Nakatsukasa, O. Sete, and L. N. Trefethen. *The AAA algorithm for rational approximation,* SIAM J. Sci. Comput., 40(3):A1494–A1522, 2018.