CD player model order reduction

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Contents

1 Introduction

Consider a linear time invariant multi-input multi-output system described by the state space equations

$$\dot{x}(t) = Ax(t) + Bu(t),$$

and

$$y(t) = Cx(t),$$

where $x(t)$ denotes the state vector of length n and $u(t), y(t)$ denote the input and output vectors of length p. The large sparse matrix A is of size $n \times n$ and B, C^T are of size $n \times p$. Block rational Krylov spaces [2] are a powerful tool for model order reduction provided a good set of poles is chosen. Here we explore different choices of poles, and compare how well the the reduced model behaves compared to the original system.

2 CD player

We start by loading the CD player problem, and constructing a function handle for the transfer function of the system:

$$H(s) = C(sI_n - A)^{-1}B.$$

For each set of poles, we construct the approximate transfer function

$$H_m(s) = C\,\boldsymbol{V}_m(sI_{ms} - \,\boldsymbol{V}_m^* A\,\boldsymbol{V}_m)^{-1}\,\boldsymbol{V}_m^* B.$$

We compare the two models by plotting the error

$$\|H(s) - H_m(s)\|_2$$

for s over the range of $i[10^0, 10^6]$.

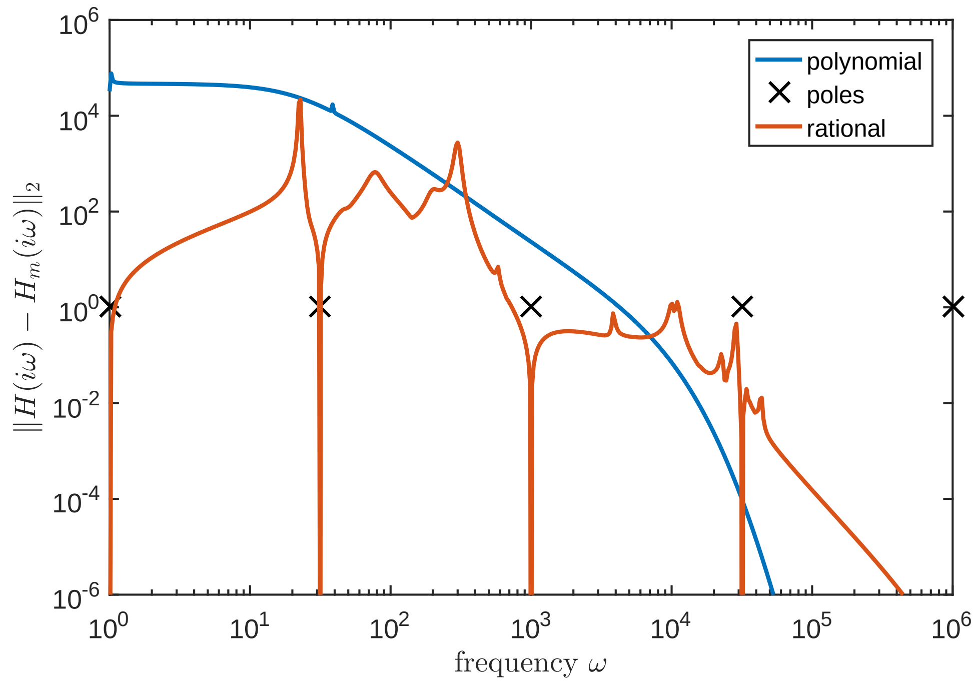
```
if exist('CDplayer.mat') ~= 2
   disp(['The required matrix for this problem can be downloaded
       from '  ...
         'http://slicot.org/20-site/126-benchmark-examples-for-
            model-reduction']);
   return
end
load CDplayer
H = @(s) C*((s*speye(size(A)) - A)\B);
```

3 Infinite poles vs equally spaced poles

We compare a block polynomial Krylov approximation against a block rational Krylov approximation. The polynomial space is constructed with 10 poles set to infinity, whereas the rational space has 5 poles logarithmically spaced in the interval $i[10^0, 10^6]$, complemented with their complex conjugates to obtain a real block rational Arnoldi decomposition. For both approaches, we plot the norm of the difference between the transfer function and the approximate tranfer function.

```
% block polynomial Krylov space
xi = inf*ones(1,10);
V = rat_krylov(A,B,xi);

Am = V'*A*V;
Cm = C*V;
Bm = V'*B;
Hm = @(s) Cm*((s*speye(size(Am)) - Am)\Bm);
Xm = @(s) V*((s*eye(size(Am)) - Am)\(V'*B));
Gam = @(s) B - (s*speye(size(A)) - A)*Xm(s);

errHm = [];
s = 1i*logspace(0,6,500);
for k = 1:length(s)
   errHm(k) = norm(H(1i*s(k)) - Hm(1i*s(k)));
end

loglog(imag(s), errHm, 'LineWidth',2); hold on
xlabel('frequency $\omega$', 'Interpreter', 'latex')
ylabel('$\|H(i \omega) - H_m(i \omega)\|_2$', 'Interpreter', '
   latex')

% block rational Krylov space
xi = 1i*logspace(0,6,5);
xi = util_cplxpair(xi, conj(xi));
```

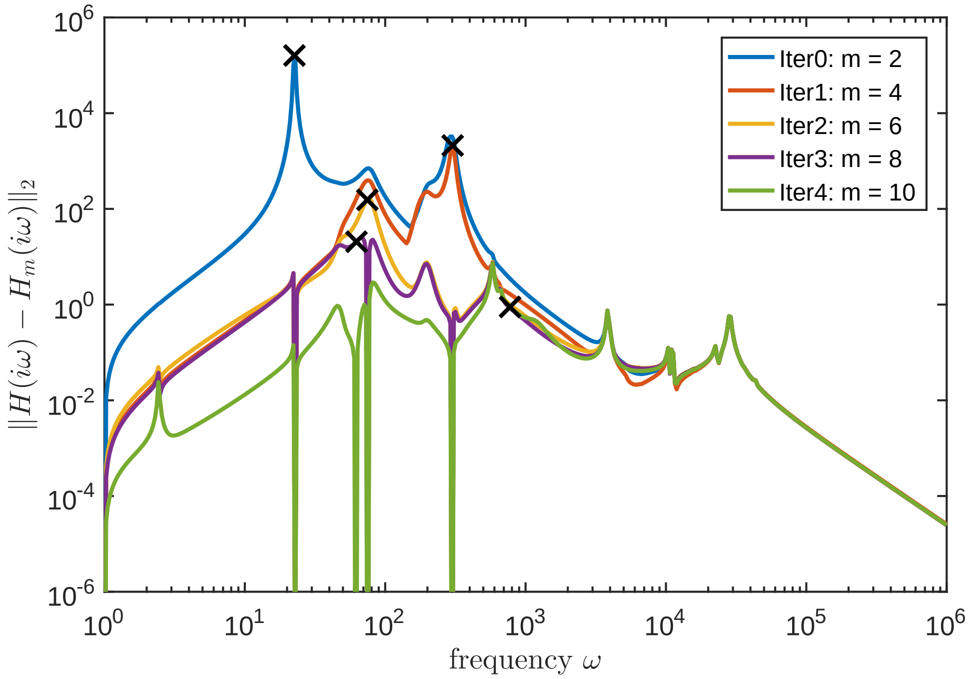
```
param.real = 1;
V = rat_krylov(A,B,xi);

Am = V'*A*V;
Cm = C*V;
Bm = V'*B;
Hm = @(s) Cm*((s*speye(size(Am)) - Am)\Bm);

errHm = [];
s = union(xi, 1i*logspace(0,6,500));
for k = 1:length(s)
    errHm(k) = norm(H(s(k)) - Hm(s(k)));
end

p = loglog(imag(xi), ones(1, length(xi)), 'kx', 'MarkerSize',
    12);
loglog(imag(s), errHm, 'LineWidth',2); axis([0, 1e6, 1e-6, 1e6])
xlabel('frequency $\omega$', 'Interpreter', 'latex')
ylabel('$\|H(i \omega) - H_m(i \omega)\|_2$', 'Interpreter', '
    latex')
legend({'polynomial', 'poles', 'rational'})
```



4 Adaptive pole selection

In this example we start with two poles at $[1i, -1i]$. We then select the following poles adaptively, in complex conjugate pairs, using the procedure described in Section 3.2 in [1]. This procedure proves to be quite effective, with the error curve dropping significantly from iteration to iteration. Note that we are using the *extension functionality* of the `rat_krylov` function, which allows to extend an existing block rational Arnoldi decomposition with new block basis vectors.

```
cand = 1i*logspace(0,6,500);
xi = [ 1i , conj(1i) ];
param.real = 1;
[V, K_, H_] = rat_krylov(A, B, xi, param); % initial run

param.extend = 2;  % allow for the iterative extension of BRAD
figure()
for iter = 1:5
    Am = V'*A*V;
    Bm = V'*B;
    Cm = C*V;
    Hm = @(s) Cm*((s*speye(size(Am)) - Am)\Bm);
    Xm = @(s) V*((s*eye(size(Am)) - Am)\(V'*B));
    Gam = @(s) B - (s*speye(size(A)) - A)*Xm(s);
    errHm = [];
    for k = 1:length(cand)
        nrmGam(k) = norm(Gam(cand(k)));
        errHm(k) = norm(H(cand(k)) - Hm(cand(k)));
    end
    mem(iter)=loglog(imag(cand),errHm, 'LineWidth',2);
    hold on, shg

    [val,ind] = max(nrmGam);
    loglog(imag(cand(ind)), errHm(ind), 'kx', 'MarkerSize', 12,
       'LineWidth', 2)
    xi = [ cand(ind) , conj(cand(ind)) ];
    [V,K_,H_] = rat_krylov(A, V, K_, H_, xi, param); % extend
       decomposition
end

legend(mem,{'Iter0: m = 2', 'Iter1: m = 4', 'Iter2: m = 6', ...
    'Iter3: m = 8', 'Iter4: m = 10'})
xlabel('frequency $\omega$', 'Interpreter', 'latex')
ylabel('$\|H(i \omega) - H_m(i \omega)\|_2$', 'Interpreter', '
   latex')
axis([0, 1e6, 1e-6, 1e6])
```



5 References

[1] O. Abidi, M. Hached, and K. Jbilou. *Adaptive rational block Arnoldi methods for model reductions in large-scale MIMO dynamical systems,* New Trends Math. Sci., 4(2):227–239, 2016

[2] S. Elsworth and S. Güttel. *The block rational Arnoldi method,* SIAM J. Matrix Anal. Appl., 41(2):365–388, 2020.