

# An overview of the example collection

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Welcome to this example collection, which intends to demonstrate some of the features of the MATLAB Rational Krylov Toolbox. Simply use the menu on the left-hand side to navigate through the collection. Each example is available as a MATLAB **m-file** and in PDF format (see the links in the above header). All examples are also included in the `rktoolbox.zip` file available from the RKToolbox website.

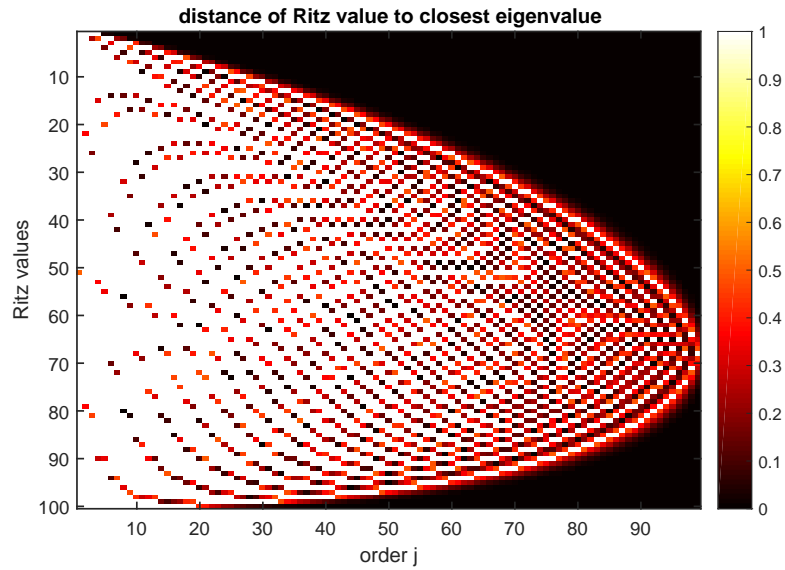
New examples will be added over time and contributions are more than welcome. If you would like to add an example to this collection please email your MATLAB file to `stefan.guettel@manchester.ac.uk`. You can use any **m-file** of this collection as a template.

Here is a simple example illustrating the fascinating convergence behaviour of rational Ritz values [1]. The matrix  $A$  is diagonal with 100 equispaced eigenvalues in the interval  $[1, 100]$ . Using the rational Arnoldi method [2,3] implemented in `rat_krylov`, we compute Ritz values associated with rational Krylov spaces of increasing dimension with poles alternating between 0 and  $\infty$ . We then visualize the distance of each Ritz value of order  $j = 1, \dots, 99$  to its closest eigenvalue:

```
N = 100; m = 99;
A = spdiags((1:m+1)', 0, N, N);
b = ones(N, 1);
xi = zeros(1, m); xi(1:2:end) = inf;

[V, K, H] = rat_krylov(A, b, xi);

Am = H(1:m, 1:m)/K(1:m, 1:m);
R = ones(N, m);
for j = 1:m
    ritz = eig(Am(1:j, 1:j));
    R(round(ritz), j) = abs(ritz - round(ritz));
end
imagesc(R); colormap(hot(100)); colorbar
xlabel('order j'); ylabel('Ritz values');
title('distance of Ritz value to closest eigenvalue')
```



- [1] B. Beckermann, S. Güttel, and R. Vandebril. *On the convergence of rational Ritz values*, SIAM J. Matrix Anal. Appl., 31(4):1740–1774, 2010.
- [2] A. Ruhe. *Rational Krylov: A practical algorithm for large sparse nonsymmetric matrix pencils*, SIAM J. Sci. Comput., 19(5):1535–1551, 1998.
- [3] A. Ruhe. *The rational Krylov algorithm for nonsymmetric eigenvalue problems. III: Complex shifts for real matrices*, BIT, 34(1):165–176, 1994.